THE

MATHEMATICAL GAZETTE.

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc., AND PROF. E. T. WHITTAKER, M.A., F.R.S.

LONDON:

G. BELL AND SONS, LTD., PORTUGAL STREET, KINGSWAY,

AND BOMBAY.

Vol. IX.	January, 1918.	No. 133.

A CENTURY AGO.

The *Annual Register* for 1816 contained a parody of a Cambridge Examination paper, intended to ridicule the classical papers set at Trinity. In the same year the following parody was circulated in the University. It was published in the *Gentleman's Magazine* for April, 1850. A few questions are omitted—*autres temps, autres mœurs.*

Utopia University, 1816.

(1) Find the value of 0, and from hence explain the general expression of a man sending a circular letter to his creditors.

(2) Construct a craniometer on the principle of the hydrometer, pointing out the uses to which such an instrument would be applicable.

(3) An orifice is cut reaching from the surface to the centre of the earth. In what time will a cub of given magnitude descend with the velocity acquired in a chace of a given number of miles?

(6) Where must an eye be placed to see distinctly the books missing from the University Library, the fountain of the Nile, and the author of these problems?

(7) Given that a man can stand 24 hours on 2 legs; show that the same man can stand 12 hours on one.

(8) Investigate an expression for the law of the centrifugal force in modern extempore discourses.

(9) To determine the least quantity of material out of which the modern dress of a fashionable female can be constructed.

(10) Prove all the roots of radical reform to be either irrational or impossible.

(11) Given the three sides of a steel triangle just immersed in sulphuric acid. Required a solution of the triangle.

(12) Compare the eccentricities of Lord Stanhope, the comet in 1811, and Sir Frederick Flood.

(13) Reconcile Hoyle and Euclid, the latter of whom defines a point to be without magnitude, the former to equal five.

(14) Sum your rental to n terms by the method of increments, your debts *ad infinitum* by the differential method.

(15) Find practically the nature and length of the lunar caustic.

(16) Seven funipendulous bodies are suspended from different points in a common system at the Old Bailey; to find the centre of oscillation.

(17) Required to find the function of a sinecure.

(20) Find the whole area of the wooden spoon, and compare that of the Holy Land with the area of that part of it generally called Clapham Common.

(21) Investigate the magnifying power of the eye of Baron Munchausen, and show that any straight line placed before it will form a conic section, no other than the common hyperbola.

(22) Construct a theorem, by the assistance of which the periodic time of *status pupillaris* may be expanded to any number of terms.

(23) In the general equation (Algebra part second) shew, that the probable reason why Wood invariably uses p and q in preference to the other letters of the alphabet may be deduced from the general expression "Mind your p's and q's."

(24) Given a Berkshire pig, a Johnian pig, and a pig of lead, to compare the respective densities.

(*To be continued.*)

---

10. **Music and Mathematics.** "Between music and mathematics there is an intimate relationship which has not perhaps been noticed. If we conceive of the general domain of ideas as forming a continuous system, the field of mathematical ideas is but a very small portion of the whole. In my opinion they figure in it as much as the Fraunhofer lines in the solar spectrum. Thus there is a mathematical scale just as there is a musical scale. From this point of view a piece of mathematical reasoning is comparable to a series of chords drawn from the intellectual lyre formed by the mathematical lines of human thought, and the discovery of a new branch of mathematics is comparable to that of a new harmonic modulation. But while it is possible to displace the musical scale without altering the harmonic relations, we cannot thus displace the scale in mathematics—at least, there is no instance of it in the history of the science—without the same theorem being presented at different periods or in different people in different *tones*. Thus the existence of mathematical chords is absolute, while that of musical chords is relative. . . ."—E. Beltrami to G. Wolff.

"Herein I think one clearly discerns the internal grounds of the coincidence or parallelism, which observation has long made familiar, between the mathematical and musical ἦθος. May not Music be described as the Mathematic of sense, Mathematic as Music of the reason? the soul of each the same! Thus the musician *feels* Mathematic, the mathematician thinks Music,—Music the dream, Mathematic the working life—each to receive its consummation from the other when the human intelligence, elevated to its perfect type, shall shine forth glorified in some future MOZART-DIRICHLET or BEETHOVEN-GAUSS—a union already not indistinctly fore-shadowed in the genius and labours of Helmholtz!"—SYLVESTER, "On the real and imaginary roots of algebraical equations," *Phil. Trans.* iii. 1864, p. 613.

"I believe that there is much that is true in Sylvester's thought. I have not profoundly studied harmony, that part of the science of music which may be regarded as a rational theory with its postulates and axioms from which all that remains can be deduced. It certainly seems to me that the mental processes involved in harmony are practically identical with those employed in mathematics. Or, using a materialistic metaphor, it may be said that in both sciences we see in action the same organs. As for Composition, in the wider sense of the word, it seems to me that other elements enter in, of a sufficiently different character from those in Harmony. However that may be, it is noteworthy that Meyerbeer,* one of the greatest harmonists and composers of modern times, began his career with the study of mathematics, to which he was devoted."—Beltrami, *Opere Matematiche di Luigi Cremona*, iii. pp. 479-481 n., 1917 (Hoepli, Milan).

---

* His brother, W. Beer, was an astronomer of note, associated in selenography with J. H. Mädler.

THE PRINCIPLES OF PROBABILITY AND APPROXIMATIONS IN ARITHMETIC.

BY W. HOPE-JONES, B.A.

(*Continued from p.* 113.)

5. To find the probability that if two quantities chosen at random are measured correct to n figures, their product so obtained is correct to n figures.

Let the quantities, after multiplication or division by an integral power of 10, so as to be brought between 1 and 10, be a and b. Then, since the frequency of X is proportional to $\frac{1}{X}$, that is $\frac{d \log X}{dX}$, all values of $\log_{10} a$ and $\log_{10} b$ from 0 to 1 are equally likely.

$\therefore$ ab is equally likely to be greater or less than 10.

$\therefore$ the required probability is the Arithmetic Mean between the probabilities when $ab > 10$ and when $ab < 10$.

Case I. $ab > 10$.

Let a and b be measured, correct to n figures, as $a + x/10^{n-1}$ and $b + y/10^{n-1}$, which are both (integers)$/10^{n-1}$, x and y being between $\pm\frac{1}{2}$.

Let $a < b$.

The chance that the product is incorrect to n figures is the chance that $\{\text{an integer} + \frac{1}{2}\}/10^{n-2}$ comes between ab and $(a + x/10^{n-1})(b + y/10^{n-1})$. Let this chance be C. Then

$$C = \text{chance that integer} + \tfrac{1}{2} \text{ comes between } 10^{n-2}ab \text{ and } 10^{n-2}ab + (bx+ay)/10 + (xy)/10^n$$

$$= \text{mean value of } ((bx+ay)/10 \sim 0), \text{ approximately,}$$

the $(xy)/10^n$ term being negligible if $n > 1$.

[*Note.* $((bx+ay)/10 \sim 0)$ is necessarily less than 1.]

Positive and negative values of $bx+ay$ being equally likely (if n is large) to cause an error in the approximate answer, only the region over which $bx+ay > 0$ need be considered.

The lower limit of b is a if $a > \sqrt{10}$, $10/a$ if $a < \sqrt{10}$.

$$\therefore \quad C = \frac{\left\{\int_{a=1}^{a=\sqrt{10}} \int_{b=10/a}^{b=10} + \int_{a=\sqrt{10}}^{a=10} \int_{b=a}^{b=10}\right\} \int_{y=-\frac{1}{2}}^{y=\frac{1}{2}} \int_{x=-ay/b}^{x=\frac{1}{2}} \left(\frac{bx+ay}{10}\right) \frac{da\, db\, dy\, dx}{ab}}{\left\{\int_{1}^{\sqrt{10}} \int_{10/a}^{10} + \int_{\sqrt{10}}^{10} \int_{a}^{10}\right\} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-ay/b}^{1} \frac{da\, db\, dy\, dx}{ab}}$$

$$= \log_{10} e - (3299 - 320\sqrt{10})(\log_{10} e)^2/1800 = (\cdot 19464) = \cdot 195 \text{ (say)}. \quad \ldots\ldots(15)$$

Case II. $ab < 10$.

As before, $a < b$ and $bx + ay > 0$. Then

$$C = \text{chance that (an integer} + \tfrac{1}{2})/10^{n-1} \text{ comes between } ab \text{ and } (a + x/10^{n-1})(b + y/10^{n-1})$$

$$= \text{mean value of } (bx+ay, \text{ counted as 1 when greater than 1})$$

$$= \frac{\int_{a=1}^{a=\sqrt{10}} \int_{b=a}^{b=10/a} \int_{y=-\frac{1}{2}}^{y=\frac{1}{2}} \int_{x=-ay/b}^{x=\frac{1}{2}} (bx+ay) \frac{da\,db\,dy\,dx}{ab} - \iiiint (bx+ay-1) \frac{da\,db\,dy\,dx}{ab}}{\int_{1}^{\sqrt{10}} \int_{a}^{10/a} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-ay/b}^{\frac{1}{2}} \frac{da\, db\, dy\, dx}{ab}}$$

the second integral in the numerator being taken over the space in which

$$bx + ay > 1. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

The denominator of (16)$=(\log_e 10)^2/8$.

The 1st integral in the numerator$=961/720-2\sqrt{10}/9=\cdot 6320$.

In the 2nd integral in the numerator, the limits of x are $(1-ay)/b$ to $\frac{1}{2}$. The upper limit of y is $\frac{1}{2}$, and the lower limit is the greater of $-\frac{1}{2}$ and $(2-b)/2a$ (x being then $\frac{1}{2}$).

If $-\frac{1}{2}>(2-b)/2a$, $\therefore\ b>a+2$; $\therefore\ 10/a>a+2$; $\therefore\ a<\sqrt{11}-1$.

$\therefore$ the lower limit of y is $(2-b)/2a$ except when $a<\sqrt{11}-1$ and $b>a+2$, in which case it is $-\frac{1}{2}$.

$\therefore$ the second integral in the numerator of (16)

$$=\left\{\int_{a=1}^{a=\sqrt{11}-1}\int_{b=a}^{b=a+2}\int_{y=(2-b)/2a}^{y=\frac{1}{2}}+\int_{1}^{\sqrt{11}-1}\int_{a+2}^{10/a}\int_{-\frac{1}{2}}^{\frac{1}{2}}+\int_{\sqrt{11}-1}^{\sqrt{10}}\int_{a}^{10/a}\int_{(2-b)/2a}^{\frac{1}{2}}\right\}\int_{x=(1-ay)/b}^{x=\frac{1}{2}}(bx+ay-1)\frac{da\,db\,dy\,dx}{ab}$$

$$=1\tfrac{217}{720}-\frac{47\sqrt{11}}{180}+\frac{\log_e 10}{120}+\frac{3\log_e 3}{16}-\frac{\log_e(\sqrt{11}-1)}{60}-\frac{(\log_e 10)^2}{16}$$
$$-\frac{\log_e 10\log_e(\sqrt{11}-1)}{4}+\frac{\{\log_e(\sqrt{11}-1)\}^2}{8}+\frac{1}{4}\int_1^{\sqrt{11}-1}\frac{\log_e(a+2)}{a}\,da$$

[$\int_1^{\sqrt{11}-1}\frac{\log_e(a+2)}{a}\,da$ cannot, so far as I know, be integrated exactly; but its approximate value is $1\cdot 0638$.]

$$\therefore\ \frac{c}{8}(\log_e 10)^2=\frac{961}{720}-\frac{2\sqrt{10}}{9}-1\tfrac{217}{720}+\frac{47\sqrt{11}}{180}-\frac{\log_e 10}{120}-\frac{3\log_e 3}{16}+\frac{\log_e(\sqrt{11}-1)}{60}$$
$$+\frac{(\log_e 10)^2}{16}+\frac{\log_e 10\log_e(\sqrt{11}-1)}{4}-\frac{\{\log_e(\sqrt{11}-1)\}^2}{8}-\frac{1\cdot 0638}{4}$$
$$=\cdot 6733. \qquad\qquad (17)$$

The Arithmetic Mean of the values of C when $ab>10$ and when $ab<10$, which are $\cdot 1946$ and $\cdot 6733$, is $\cdot 434$.

$\therefore$ the probability that, if two quantities are measured correct to n figures and multiplied, their product so obtained is correct to n figures, tends, as n increases, to the limit $\cdot 566$, which is a good approximation to the exact value if $n>1$. .... (18)

6. When the area of a closed curve is estimated by counting squares, there are various ways of dealing with the squares which are partly in and partly out. The most usual are:

(1) To count them all as $\frac{1}{2}$ each.

(2) To count those more than $\frac{1}{2}$ in (called for short "inners") as 1, and those less than $\frac{1}{2}$ in ("outers") as 0.

(3) To count each inner as $\frac{3}{4}$, each outer as $\frac{1}{4}$.

The last two may be modified by taking as $\frac{1}{2}$ any square so nearly bisected that there is any difficulty in deciding whether it is an inner or an outer.

It is required to compare the probable errors in these methods.

Let the squares be so small in comparison with the dimensions of the curve that its curvature while crossing one square may be neglected.

Take as unit the length of the side of a square. Let θ be the smallest angle between the curve, where it cuts some particular square, and a side of the square. Let this side be AB.

Then $\theta \leqq \frac{\pi}{4}$.

The frequency of θ is proportional to the breadth of the target that the diagonal AC presents to the curve, that is, to $\sin\theta+\cos\theta$. For any particular value of θ, the curve is equally likely to cut CF at any point between C and F.

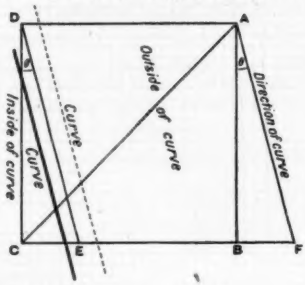
Let it cut CF at a distance a from C.

If $1>a>\tan\theta$, as in the dotted curve, the area enclosed (x) is

$$\tfrac{1}{2}\tan\theta+(a-\tan\theta)\,;\quad\therefore\ da=dx.$$

$\therefore$ the probability that the area enclosed is between x and $x+dx$ = the probability that the intercept on CF is between a and $a+da$, which

$$=da/CF=dx/(1+\tan\theta).$$



If $a<\tan\theta$, as in the thick curve,

$$x=\tfrac{1}{2}a^2\cot\theta\,;\quad\therefore\ a=\sqrt{2x\tan\theta}.$$

$\therefore$ the probability that the area enclosed is between x and $x+dx$ = the probability that the intercept on CF is between a and $a+da$, which

$$=da/CF=dx/(1+\tan\theta)\times\sqrt{\tan\theta/2x}.$$

$\therefore$ when θ may have any value from 0 to $\frac{\pi}{4}$ (its frequency being $\sin\theta+\cos\theta$), the probability of any area between $x(<\frac{1}{2})$ and $x+dx$ is

$$dx\times\frac{\displaystyle\int_0^{\tan^{-1}2x}\frac{\{(\sin\theta+\cos\theta)\,d\theta\}}{(1+\tan\theta)}+\int_{\tan^{-1}2x}^{\frac{\pi}{4}}\sqrt{\frac{\tan\theta}{2x}}\,\frac{(\sin\theta+\cos\theta)\,d\theta}{1+\tan\theta}}{\displaystyle\int_0^{\frac{\pi}{4}}(\sin\theta+\cos\theta)\,d\theta}\,;$$

$$\therefore\ (\text{probability})/dx=2x/\sqrt{4x^2+1}+1/\sqrt{2x}\times\int_{\tan^{-1}2x}^{\frac{\pi}{4}}\sqrt{\sin\theta\cos\theta}\,dx. \quad (19)$$

This is the frequency of x if $x<\frac{1}{2}$: if $x>\frac{1}{2}$, its frequency is evidently the same as that of $1-x$.

When $x=1/2$, the 2nd term $=0$, and the frequency is $1/\sqrt{2}$.

Let a closed curve pass through n squares, enclosing

a fraction $>\frac{1}{2}+p$ of $(n-s)/2+y$ squares,
a fraction $<\frac{1}{2}-p$ of $(n-s)/2-y$ squares,
and a fraction between $\frac{1}{2}\pm p$ of s squares,

p being a small quantity whose square may be neglected.

Any square outside the limits $\frac{1}{2}\pm p$ is easily recognizable as an inner or an outer: between $\frac{1}{2}\pm p$ it is taken as $\frac{1}{2}$.

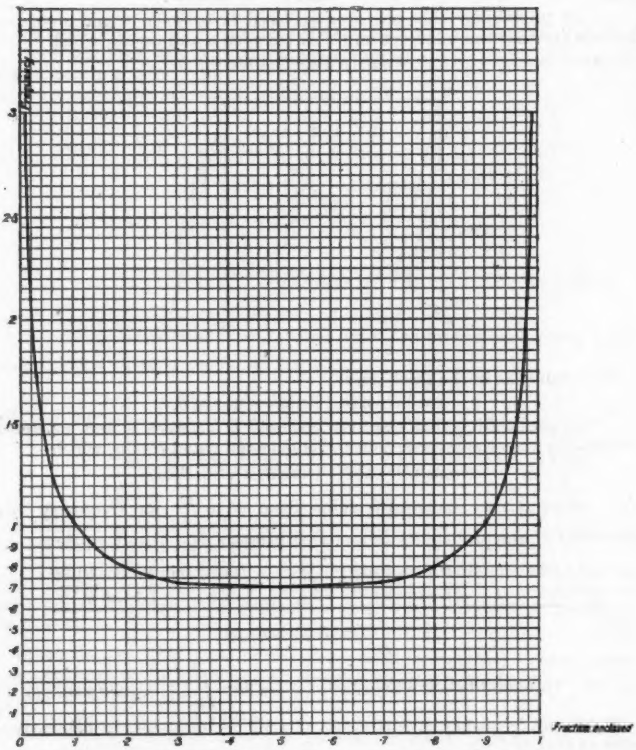
If a very large number of "obvious inners," that is, fractions $>\frac{1}{2}+p$, are distributed according to the frequency already found, their mean, and their mean square of deviation from that mean, will each converge towards a definite limit.

Let these limits be called $\frac{1}{2}+B$ and δ^2.

For obvious outers the corresponding quantities are $\frac{1}{2}-B$ and δ^2. We may call $\frac{1}{2}+B$ and $\frac{1}{2}-B$ the "regional means."

Then the real area of all the squares partly enclosed

$$=\frac{n}{2}+\text{sum of all the excesses over } \tfrac{1}{2}.$$



Graph of frequency of fractions of square enclosed by curve.

In the central region the sum of the excesses over $\frac{1}{2}$ is of the order p^2, and is omitted.

$$\therefore \text{ area}=\frac{n}{2}+B\left(\frac{n-s}{2}+y\right)-B\left(\frac{n-s}{2}-y\right)+\text{sum of excesses over regional means.}$$

$$=\frac{n}{2}+2By+\text{sum of excesses over regional means.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

All the approximate methods mentioned are special cases of taking the area as

$$\frac{h}{h+j} \text{ number of inners}$$

$$+\frac{j}{h+j} \quad \text{,,} \quad \text{,,} \quad \text{outers}$$

$$+\tfrac{1}{2} \quad \text{,,} \quad \text{,,} \quad \text{indistinguishables from } \tfrac{1}{2}.$$

In 1st method, $h=j$; in 2nd method, $j=0$; in 3rd method, $h=3j$.
The approximate area

$$=h/(h+j)\times\{(n-s)/2+y\}+\{j/(h+j)\}\times\{(n-s)/2-y\}+\frac{s}{2}$$
$$=n/2+y(h-j)/(h+j);$$

$\therefore$ the error $=y\{(h-j)/(h+j)-2B\}-$ sum of excesses over regional means.

The mean value of this sum $=0$, and the mean value of its square $=(n-s)\delta^2$ there being $n-s$ obvious inners or outers under consideration.

$$\therefore \text{ mean (error)}^2=y^2\{(h-j)/(h+j)-2B\}^2+(n-s)\delta^2. \quad\text{............(21)}$$

To make this a minimum, we must make $h-j=2B(h+j)$.

B depends on p; but from the graph, or by approximate integration, it is found when $p=0$ to be about ·3, from which value it cannot deviate much, p being small. $\quad \therefore\ h-j=\cdot6(h+j);\quad \therefore\ h=4j.$

$\therefore$ the most accurate way of estimating the area of the squares partly enclosed is to add together

$\frac{1}{2}$ the number of squares neither obviously more nor less than half in,
$\frac{4}{5}$ of " " " obviously more than half in,
and $\frac{1}{5}$ " " " " less " " .... (22)

Of the other methods,

the first is the least accurate, making $(\lambda-2B)^2=(-\cdot6)^2$,
the second is more accurate, making $(\lambda-2B)^2=(\cdot4)^2$,
the third is the most accurate, making $(\lambda-2B)^2=(\cdot1)^2$,

where $\lambda=(h-j)/(h+j)$.

The mean value of $4y^2$ is $(n-s)$, the number of squares which are distributed over two equally likely regions.

The frequency when $x=\frac{1}{2}$ being $\frac{1}{\sqrt{2}}$, the mean value of s is $np\sqrt{2}$: and by the ordinary method of finding the Centre of Gravity of a body made up of two parts, B is found to be $A(1+p\sqrt{2})$, A being the value of B when $p=0$, that is, the mean excess over $\frac{1}{2}$, necessarily positive, of an infinite number of inners.

By the ordinary method of finding the Moment of Inertia of a body made up of two parts, δ^2 is found to be $\Delta^2-p\sqrt{2}(A^2-\Delta^2)$, Δ being the value of δ when $p=0$. $[\Delta^2=\text{about }\cdot03.]$

$\therefore$ the mean value of the mean (error)^2, which is $y^2(\lambda-2B)^2+(n-s)\delta^2$, is

$$(n-np\sqrt{2})\{\tfrac{1}{4}(\lambda-2A-2Ap\sqrt{2})^2+\Delta^2-p\sqrt{2}(A^2-\Delta^2)\}$$
$$=n\left[\tfrac{1}{4}(\lambda-2A)^2+\Delta^2-\frac{p}{2\sqrt{2}}(\lambda^2)\right],$$

where $\lambda=(h-j)/(h+j)$. .... (23)

This mean (error)^2, and the probable error, which is ·6745 of its square root, decrease as p increases, giving the paradox that, the finer our perception of whether a nearly bisected square is more or less than half included, the less accurate will our answer be. But in practice it is well not to extend too widely the limits between which squares are classified as "indistinguishable from $\frac{1}{2}$," firstly, because the powers of p have been neglected, and secondly, because it would be practically impossible to keep the limit constant, below which a square is to be counted an outer, unless this "nearly bisected" classification is reserved for squares of which it is really doubtful whether more or less than half is included.

When $h=4j$, and p is small, the probable error is about $\sqrt{n}/10$.

Eton College. W. Hope-Jones, Lieut. R.A.

SOME INCIDENTAL WRITINGS BY DE MORGAN.

(*Continued from p.* 122.)

XV.

I. ii. 133. **The Geometrical Foot.**—In several places I have discussed the existence and length of what the mathematicians of the sixteenth century *used*, and those of the seventeenth *talked about*, under the name of the *geometrical foot*, of four palms and sixteen digits. (See the *Philosophical Magazine* from December, 1841, to May, 1842; the *Penny Cyclopaedia*, "Weights and Measures," pp. 197, 198; and *Arithmetical Books*, &c., pp. 5-9.) Various works give a figured length of this foot, whole, or in halves, according as the page will permit; usually making it (before the shrinkage of the paper is allowed for) a very little less than $9\frac{3}{4}$ inches English. The works in which I have as yet found it are Reisch, *Margarita Philosophica*, 1508; Stöffler's *Elucidatio Astrolabii*, 1524; Fernel's *Monalosphaerium*, 1526; Köbel, *Astrolabii Declaratio*, 1552; Ramus, *Geometria*, 1569 and 1580; Ryff, *Quaestiones Geometricae*, 1621. Query. In what other works of the sixteenth, or early in the seventeenth century is this foot of palms and digits to be found, figured in length? What are their titles? What the several lengths of the foot, half foot, or palm, within the twentieth of an inch? Are the divisions into palms or digits given; and, if so, are they accurate subdivisions? Of the six names above mentioned, the three who are by far the best known are Stöffler, Fernel, and Ramus; and it so happens that their subdivisions are *much* more correct than those of the other three, and their whole lengths more accordant.

A. De Morgan.

[Reisch, Gregorius. Margarita Philosophica Noua, *cui insunt sequentia. Epigrammata in comēdationem operis. Institutio Grammaticę Latinę. Pręcepta Logices. Rhetoricę informatio. Ars Memorandi Ravennatis. Beroaldi modus cōponēdi Epi.* Arithmetica. Musica Plana. Geometrie Principia. Astronomia, *cum quibusdā de Astrologia.* Philosophia Naturalis. *Moralis Philosophia cū figuris*, 4to, 1512, Argentinae, Joa. Grüninger. This, says S., is the fifth edition. It contains an *Appēdix Matheseos*, pp. 128, which includes treatises on the atrolabe, perspective, etc. He gives the known editions as those of 1496, 1503, 1508, 1512, 1515, 1517, 1535, 1583, and 1599.

The following is from De Morgan's *Arithmetical Books*, p. 4: He examined the edition printed at Basle in 1508. "According to Kloss (from his own copies) the first edition of this curious book is *Friberg* fifteen-three, the second and third both *Strasburg* fifteen-four (one printed by Gruninger, which I have seen, and one by Schott), and he calls the one before me the fourth. But Hain marks as the first edition one which appears to be printed at Heidelberg, fourteen-ninety-six, *quarto*. The one before me is not the third; it is evident that a former title-page has been reprinted: for though this edition is printed (as appears from the colophon) by Furter and Scotus of Basle, the title-page bears 'Jo. Schottus Argent. lectori S.,' which agrees with Kloss's statement that the one before this was printed by Schott of Strasburg. The Arithmetic, which is a part of the system of philosophy here laid down, has a frontispiece representing Boethius at one table, with Arabic numerals before him; and Pythagoras at another with counters. Pythagoras among the Greeks, Apuleius and Boethius among the Romans, were often made the inventors of Arithmetic. The arithmetic is divided into speculative and practical. The former is a summary of Boethius, often in the words of John de Muris. The latter is a short treatise on *Algorithm*, as it was called, or the rules of computation by the Arabic numerals. There is also computation by counters, fractions common and sexagesimal, and the rule of three. Many works of fifty years later do no more difficult questions. That Boethius was the author of the Arabic numerals was a common notion at the time, revived in our own day. I have seen another edition of *Strasburg*, fifteen-twelve; and there is said to

be another edition by Orontius Fineus, *Paris* (?) fifteen-twenty. There is also an Italian translation, *Venice*, fifteen-ninety-nine, *quarto*, with the additions of Orontius, translated by Giovan Paolo Galucci.

"If the number were sufficient of those who wish to take their notions of liberal education in Europe at the time immediately preceding the Reformation from original sources, and not from the reports of others, a reprint of the *Margarita Philosophica* would be made. The diversity of the matters of which it treats, and the largeness of its circulation, stamp it as the best book for such a purpose.

"The *Margarita Philosophica* is the earliest work I have found in which mention is made of that peculiar system of measures which was current among the mathematicians of the sixteenth century, and which has caused no little confusion among writers on metrology. I have already given some account of this ill-understood system, and shall here endeavour to present the whole case with some additional evidence.

"The Roman foot (of 11·62 inches English) was of course established throughout the empire; and with it the Roman pace * of five feet, or 58·1 inches or 4·84 feet English. The natural pace of a man in our day is as nearly as possible five English feet: Paucton's experiments on the walking paces of individuals gave him 59·7 inches English, or 4·98 feet. In the British army the step, both what is called the ordinary step and the quick step, is, by regulation, thirty inches: making a pace of five feet. The Roman pace, by which distances were actually measured, was that of a soldier on the march: and, as might be expected, the weight of his arms and other equipage seems to have shortened his pace a little. But so near were these measures, as actually used by the Romans, to the natural ones from which they derived their names, that it was customary, not only to recur to legal standards, but, in the absence of ready access to them, to make use of the natural foot and pace. And we also know from Roman writers, that a somewhat fanciful relation, though not very far from the truth, was established between the breadth of the hand across the middle of the fingers, the *palm*, and the length of the *foot*. It was taken that four palms made a foot; and a palm was made of course to consist of four (average) finger-*breadths* or digits. This division into palms and digits was the most recognised division of the Roman foot: that into inches, or *unciae*, is well known not to belong to the foot merely, but to anything else. Whatever magnitude was called unity, the *uncia* was its twelfth part. Accordingly, all the Roman foot-rules which have been found in ruins or excavations, have the digital division, to which *some* (most, I believe) have the uncial division superadded. All this I take to be too well established to require the citation of any authorities.

"It is quite out of my power, or, as far as I know, of that of any one else, to trace the gradual alteration of the foot in different countries. It does not appear that any means were taken to institute comparisons of various measures, to be preserved as public records. Such means could have been found: that which was then the common church of Christendom might have easily regulated the weights and measures of Europe, even without appearing to do so. But in all probability, the extent of the variations was not well known until it was too late. We know, however, as a fact, that the geometers were successful in establishing a measure among themselves, and communicating it through Europe on paper. This measure, I have no doubt, they believed to be the true Roman foot: for they divide it in Roman denominations, make use of it in their quotations from Roman authors, and never hint at their having any other notion of a Roman foot. And moreover, the writers who, in the fifteenth century, recovered the true Roman foot, never mention any peculiar geometrical foot in use among mathematicians, or in any way distinguish the latter from the *wrong* Roman foot which they were correcting. That the geometers believed the Roman foot, that is their own foot, to be the *human* foot, might be easily proved. And, with such belief, they would make their so-called Roman foot too short. From a hundred measures of the feet

* The pace is derived from the double step, being the distance from the extremity of the heel at the place from which it is removed in walking to the same at the place in which it is set down again.

of adult men, furnished me by a boot-maker, and taken as they came in his books, I find the average length of the Englishman's foot to be 10·26 inches, in our day : or an inch and a third shorter than the Roman lineal measure.

"This *geometrical foot* of the mathematicians is, I make no doubt, the geometrical foot to which writers of the seventeenth century refer, or mean to refer. But, not long after the true restoration of the ancient measures, there arose a disposition among those who inquired into the subject to seek a mystical origin of weights and measures, on the supposition of some body of exact science once existing, but now only seen in its vestiges : a disposition which is not yet entirely extinct. Some speculated on the pyramids of Egypt, and tried to establish that the intention of building those great masses was that a record of measures founded on the most exact principles might exist for ever. But more turned their attention to the measurement of the earth, and, by assuming nothing more difficult than that a degree of the meridian a thousand times more accurate than that of Eratosthenes was in existence hundreds, if not thousands, of years before him, it was easy enough to make out that the whole system of Greek, Roman, Asiatic, Egyptian, etc., measures was a tradition from, or a corruption of, this venerable piece of lost geodesy. There runs through all these national systems a certain resemblance in the measures of length : if a bundle of faggots were made of foot rules, one from every nation ancient and modern, there would not be any very unreasonable difference in the lengths of the sticks.

"The metrologists who treat this subject handle it according to their several theories. Those who have none in particular either neglect it altogether, or speak of its length as uncertain, or define, with Dr. Bernard, the geometrical pace as being five feet of its own kind, without saying what this kind is. Those who have the notion of the old measure of the meridian accommodate it to their supposed ancient measure ; but at the same time, those of most research and note make it *less* than their Roman foot. Thus Paucton makes it nine-tenths of the Roman foot, which, with his version of that measure, is 10·9 inches English, and with the true one, 10·5 of the same. Similarly, Romé de L'Isle makes it more than half an inch (French) less than *his* Roman foot. As they do not refer to the geometers of the middle ages, I cannot guess whence they get their notion, otherwise than from their theory.

"I now proceed to demonstrate the existence of this geometrical foot, which I believe to have been the effort of mathematicians to perpetuate and make common what they took to be the Roman foot, on the supposition that it was nothing but the average length of the *human foot*.

"Passing over the general expressions of writers who refer to the use of the parts of the body in measurement, and who sometimes distinctly state that the determination of the human foot is necessarily that of the Roman measure, I take first the statement of Clavius, whose term of active life was the latter half of the sixteenth century and who says * very distinctly that the mathematicians, to avoid the diversity of national measures, had laid down a system for themselves. The table of measures which he gives (and dozens of other writers before him) is as follows :

1 breadth of a barleycorn.
$4 =$ 1 digit.
$16 = 4 =$ 1 palm (across the middle of the fingers).
$64 = 16 = 4 =$ 1 foot.
$96 = 24 = 6 = 1\frac{1}{2} =$ 1 cubit.
$160 = 40 = 10 = 2\frac{1}{2} = 1\frac{2}{3} =$ 1 step (gressus).
$320 = 80 = 20 = 5 = 3\frac{1}{3} = 2 =$ 1 pace.
$640 = 160 = 40 = 10 = 6\frac{2}{3} = 4 = 2 =$ 1 perch (pertica).
125 paces 1 Italic stadium.
3 stadia 1 Italic mile.
4 Italic miles a German mile
5 Italic miles a Swiss mile.

(*To be continued.*)

* "Enumerandae sunt mensurae quibus mathematici, maxime geometrae, utuntur. Mathematici enim, ne confusio oriretur ob diversitatem mensurarum in variis regionibus (quaelibet namque regio proprias habet propemodum mensuras), utiliter excogitarunt quasdem mensuras, quae certae ac ratae apud omnes nationes haberentur."—*Comm. in Sacroboscum.*

MATHEMATICAL NOTES.

524. [D.] *Note on Functionality of a Complex Variable.*

It seems very natural to assume that a functional formula, which involves two real variables x and y only in the combination $x+iy$, must necessarily represent a function of the complex variable $z=x+iy$. It may therefore be interesting to notice a particular example in which this is not the case.

In the plane of a complex variable $\zeta=\xi+i\eta$, consider the area-integral

$$u+iv=\int\frac{f(\xi,\eta)}{z-\zeta}\,dS,$$

taken over an area which includes the point $\zeta=z$ and is bounded by a contour A, f being a real function of ξ and η not involving x and y as parameters. The conditions that $u+iv$ should be a function of z are that $\partial u/\partial x-\partial v/\partial y$ and $\partial u/\partial y+\partial v/\partial x$ should both be zero. These certainly would be zero if the real and imaginary parts of the integral could be differentiated by the rule of "differentiation under the sign of integration"; but, in this instance, that rule would lead to semi-convergent integrals and is not applicable.

The differential coefficients of u and v with respect to x and y may be got by the method which is explained in Sections V. and VI. of the *Cambridge Tract* on "Volume and Surface Integrals used in Physics." The resulting formulae are in terms of limits with respect to an infinitesimal cavity round the point $\zeta=z$, whose contour may be denoted by ϵ, the direction-cosines of an inward normal to ϵ being called l, m.

It is found that

$$\frac{\partial u}{\partial x}=\operatorname*{Lim}_{\epsilon\to 0}\left[\int_\epsilon\frac{lf(\xi,\eta)(x-\xi)ds}{(x-\xi)^2+(y-\eta)^2}+\int_\epsilon^A f(\xi,\eta)\frac{\partial}{\partial x}\left\{\frac{x-\xi}{(x-\xi)^2+(y-\eta)^2}\right\}dS\right],$$

$$\frac{\partial v}{\partial y}=-\operatorname*{Lim}_{\epsilon\to 0}\left[\int_\epsilon\frac{mf(\xi,\eta)(y-\eta)ds}{(x-\xi)^2+(y-\eta)^2}+\int_\epsilon^A f(\xi,\eta)\frac{\partial}{\partial y}\left\{\frac{y-\eta}{(x-\xi)^2+(y-\eta)^2}\right\}dS\right],$$

where the first integral in each bracket is a line-integral taken round the contour ϵ, and the surface-integrals are over an area bounded internally by ϵ. No one of the above four integrals has a limit which is independent of the shape with which the cavity ϵ vanishes; but they are so paired that $\partial u/\partial x$ and $\partial v/\partial y$ are nevertheless perfectly definite and independent of the vanishing shape of ϵ. From these expressions it follows that

$$\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=\operatorname*{Lim}_{\epsilon\to 0}\int_\epsilon\frac{f(\xi,\eta)\{l(x-\xi)+m(y-\eta)\}ds}{(x-\xi)^2+(y-\eta)^2}.$$

This limit is independent of the vanishing shape of ϵ; by considering the case of a cavity in the form of a circle with centre at z, it is seen that the limit is $2\pi f(x,y)$.

Hence $\partial u/\partial x-\partial v/\partial y=2\pi f(x,y)$, and $u+iv$, in spite of appearances, is *not* a function of z.

The values of $\partial v/\partial x$ and $\partial u/\partial y$ are easily obtainable, but need not be set out here. It is found that $\partial u/\partial y+\partial v/\partial x=0$.

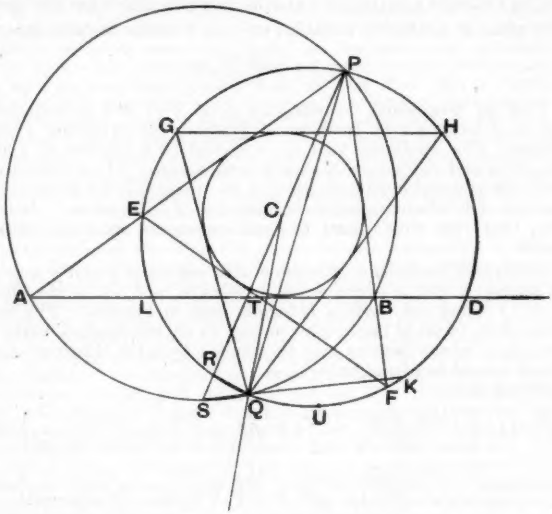
These results may be tested by considering the particular case in which A is a circle with centre at the origin of ζ, and $f(\xi,\eta)$ is a constant. The corresponding values of the integrals are well known, namely $u=\pi fx$, $v=-\pi fy$.

J. G. Leathem.

5th September, 1917.

525. [K. 1. a.] *A Theorem on Simson Lines.*

Let $PETB$ be a quadrilateral circumscribing circle (C) with centre C, and let the quad. be completed and circles (R_1) and (R_2), having radii R_1 and R_2 respectively, be described about $\triangle$s PAB and PEF and meeting again in Q. Draw PC and produce to meet the circles again in R and S. Join PQ, CQ, RQ and SQ, producing SQ to meet the $\odot\,(R_2)$ in K. Draw also the in- and circum-triangle QGH in $\odot\,(R_2)$.



Since the circles (R_1) and (C) are the circum- and in- $\odot$s of a $\triangle$ for any chord through C, the rectangle of its segments $=R_1^2-\triangle^2=2R_1r$ (since $\triangle^2=R_1^2-2R_1r$). Similarly for $\odot$ R_2. Hence

$$PC\,.\,CR=2R_2r \quad \text{and} \quad PC\,.\,CS=2R_1r.$$

Hence $CR/CS=R_2/R_1$, and as RQ and SQ subtend the same angle at P, we have also $RQ/SQ=R_2/R_1$. Therefore CQ is the external bisector of angle RQS, and the internal bisector of RQK. Hence arc $KH=$ arc RG. But angle RQS, being angle between two corresponding chords, $=$ the angle between the $\odot$s. And if U is the extremity of a diam. of (R_2) parallel to that of R_1 through S, S and U subtend at P an angle $=$ half the angle between the $\odot$s (by the ordinary theory of intersecting $\odot$s). Therefore U is the mid-point arc RK, and since S is part of diameter of (R_1) perp. to AB, U is foot of diam. of (R_2) perp. to same line; hence arc $UD=$ arc UL, or arc $KD=$ arc RL. But arc $KH=$ arc RG; therefore arc $DH=$ arc LG, and AB is parallel to GH.

Hence, if in the circle (R_2) and about the circle (C), *any* triangle PEF be described, and the point Q on circle (R_2) be fixed, the feet of the perpendiculars from Q upon the sides of this triangle are collinear with the foot of the perpendicular from Q upon the line AB, which, being a tangent to (C) parallel to the fixed tangent GH, is itself fixed. Therefore the Simson lines of a series of triangles in- and circum- to a fixed pair of $\odot$s, when taken with regard to the fixed origin Q, all pass through a point. H. RIDDELL.

REVIEWS.

Elliptic Integrals. By HARRIS HANCOCK. Pp. 104. Price $1.25 or 6s. net. 1917. (New York: John Wiley and Sons; London: Chapman and Hall, Ltd.)

This is No. 18 of the "Mathematical Monographs" edited by Merriman and Woodward, and will be found an exceedingly useful introduction to what is a rather attractive subject to a student who is interested in the technical work of integration. The volume relates almost entirely to the three well-known elliptic integrals, with tables and examples showing practical applications, and, to keep the work within limits desired by the editors, the subject is what is known as the Legendre-Jacobi theory, and the discussion is almost entirely confined to the elliptic integrals of the first and second kinds (p. 5). There is a short account of the elliptic functions which emphasises their doubly periodic properties, but there is no account of the derivation of the addition-theorem for elliptic integrals. "The underlying theory, the philosophy of the subject, I have attempted to give in my larger work on elliptic functions, vol. i." (p. 6). PHILIP E. B. JOURDAIN.

Functions of a Complex Variable. By T. M. MACROBERT. Pp. xiv+298. 12s. net. 1917. (Macmillan.)

The theory of functions of a complex variable embraces so large a field that it is not surprising to find great diversity in the text-books and treatises that deal with it. Durège, Forsyth, and Kowalewski, for example, present very different treatments of the same topic, the emphasis of the author's preference for a particular aspect of the subject being more marked in the smaller works. Mr. Macrobert's aim is to cover a wide range in a book of very moderate dimensions; he thus meets an obvious need, but inevitably sacrifices the interest that would attach to a study from some specially selected standpoint. The book will be very useful to the student who has to be economical in the apportionment of his time to various claims; but it is not likely, of itself, to grip the imagination and kindle enthusiasm.

It may arise merely from personal predilection that one feels disappointment at the brief treatment of the geometrical representation of functions. Apart from the importance of its applications, there is strong appeal to the imagination in the theory of conformal transformation; and for some there is fascination in the Riemann representation of multiform functions. Yet the Riemann surface is not mentioned throughout the work.

But if there had been more geometry there would not have been room for the theory of analytic functions, the convergence and uniform convergence of series, the gamma functions, elliptic functions, linear differential equations, hypergeometric, Legendre and Bessel functions, all of which meet with adequate clear and well arranged treatment. In preferring analysis to geometry the author is not out of the fashion.

Generally the exposition is lucid and succinct. Perhaps one may be allowed to cavil at the treatment of infinity. When, after distinguishing between the value of $f(z)$ at a point z_1 and the limit of $f(z)$ for $z \to z_1$, an illustration of discontinuity is introduced by the words "if $f(z_1)$ is infinite," one feels that there is confusion, what is meant being "if $f(z)$ becomes infinite as $z \to z_1$"; but few writers handle infinity with the precision of language that is desirable in the interests of the student. This instance is given, not as a type of Mr. Macrobert's style, but as an exception to the general carefulness of his phraseology.

The book contains a large number of illustrative examples, some fully worked out, others left to be worked by the reader. These are well selected, and will be valuable to both teacher and student. Indeed, general utility is the outstanding feature of the work. By careful arrangement and by adapting his exposition to the needs of the beginner in the subject, the author enables such a reader with the minimum of difficulty to become acquainted with the outstanding features of a wide region of new and interesting mathe-

matical thought. Mr. Macrobert is particularly to be congratulated on his success in combining comprehensiveness and brevity without sacrifice of clearness. J. G. LEATHEM.

Elementary Science for Engineering Apprentices. By W. M'BRETNEY. Pp. v+74. 1s. 1917. (Longmans, Green.)

This covers the Science syllabus usually followed in the Second Year Preliminary Technical Course, and is designed to help (*a*) the secondary or elementary school teachers who are employed to take apprentices in evening classes, and who may be unfamiliar with the practical applications; (*b*) the engineering draughtsman who has undertaken the same work but is not a teacher by experience and training; and (*c*) the apprentices. To contrive this triple debt to pay is not an easy task, but it seems to us that the author's effort is quite creditable, and that this little book probably gives just what is needed for the purpose.

The Art of Teaching Arithmetic. By J. B. THOMSON. Pp. viii+295. 4s. 6d. (Longmans, Green.)

Miss Thomson lives in a land where "teachers" are yet to be found who conceive it their duty to cane a child who produces a wrong answer to a sum. To pass from the stifling atmosphere, connoted by such an attitude towards the child mind, to the fresh breezes that make their influence felt in most pages of her book is to cross a gulf of ignorance and prejudice, a gulf of terror and of tears. The book is clearly the outcome of patient thought, of experience, and of psychological insight. It brims with sympathy for the budding mind ever brought face to face with new problems, and for the teacher—often temporarily helpless in the simultaneous task of dealing with flesh and blood, of devising ways and means of explaining difficulties, of creating and sustaining interest, and always subject to the descent from "above" of authority that is not always discriminating or even always intelligent. The book, says Miss Thomson, is not for the mathematical specialist. With all respect we venture to say that it is distinctly a book for the mathematical specialist who has to deal with the infant mind. He may wish to, or indeed may have to, teach his own children, and at any rate he will desire to know if they are being taught upon wise and well-thought out principles. He may blush when he sees Miss Thomson asking for decimals like £3.48527. He may wonder why (p. 161) in finding the H.C.F. of 96 and 128 she has not taken her class on a game of hide and seek for the factors that are ever so cunningly concealed in these numbers and also in $128-96$, *i.e.* in 32 (the "still there" game); or if in 57 and 15, these "still there" in $4\times 15-57$, *i.e.* in $60-37$, *i.e.* in 3, and surely when you know your prisoner is in 3, you can round him up swiftly and deftly. He may shudder at the old-fashioned methods for L.C.M. as well as H.C.F., methods that (*v.* top of p. 163 and foot of p. 164) most surely mask what is really being done in the processes. But apart from such blemishes he will find many things of which he has never thought, and of which he could not be expected to think unless circumstances had brought him face to face with them.

We may not agree on all points with the author, but our differences are not differences of principle. The right spirit informs her work, and the wider spread of such volumes as these must lead to the healthy differences of opinion which ensure progress in the one direction we care about—that its rightful heritage of sane instruction shall be secured to the youth of our land.

Mathematical Papers for Admission into the R.M. Academy and the R.M. College. Feb.-July, 1917. Edited by R. M. MILNE. Pp. 30. 1s. 3d. 1917. (Macmillan.)

The regular appearance of this collection is a matter of satisfaction to teachers in Army and other classes. The answers to each paper are appended.

The Principles of Rational Education. By C. A. MERCIER. Pp. 87. 2s. 9d. net. 1917. (Mental Culture Enterprise, 329 High Holborn, W.C. 1.)

Dr. Mercier's contribution to the great controversy on education is like everything else from his pen—trenchant, informing, and stimulating. He

tells us roundly that ten years of school life in these islands leave the mind of the student a blank. There are no qualifications. "They know nothing at all." The difficult task of discovering entirely useless subjects for a school curriculum has been achieved with marvellous success. Can we match the uselessness of knowing names of rulers, the order of their succession, the wars they fought in, the rivals they murdered, the dates of their battles? "Yes. We can teach our scholars the mystical speculations of muddle-headed men who have written unintelligible nebulosity . . . called Moral and Metaphysical Philosophy; and . . . a subject which even a teacher of it calls a silly game, and teach it under the imposing name of Logic."

He places music, chess and mathematics in the same group of isolated faculties, and states that all three alike have no influence, benign or baleful, upon the cultivation of other forms of ability. Mathematical ability is largely recreative, has in most cases little survival value; its utility is out of proportion to the labour involved; it is as liable to mislead as to lead aright. Argument on these points is wasted (pp. 16-19).

Among the points in which we are heartily at one with Dr. Mercier is the prominence he awards to the knowledge and use of the mother tongue. The child who is compelled, from his Arithmetic papers onward, to state in words his argument as he proceeds from step to step, is thereby compelled to construct sentences that mean one thing and only one thing, and the more clearly he understands what he is doing the less likely he is to find difficulty in accurately stating what he is doing. The effort to express is accompanied by reference to the nature of the thought to be expressed—the reciprocal check is continuous, and is beneficial to both processes.

Harrap's Introductory Algebra. By W. Farquharson. Pp. 141. 1s. 6d. net. 1917. (G. G. Harrap & Co.)

This little book is to be commended for the question-and-answer method in dealing with new ideas in the early stages of the work. "This Socratic method is not intended to supplant, but simply to guide and supplement, the work of oral teaching, and to preserve, as it were, a permanent record of it on certain critical points for repeated reference." We object to its "rules." It is true that the author hopes "that they will have to be applied merely as convenient summaries of fully realised knowledge." But they are set forth in all the glories of black type. We venture to believe that the only students who have recourse to the "rules" will be the lazy and indifferent. And there they will sometimes meet with what they deserve. Consider the *Rule* on p. 55 as a "convenient summary"—"Take all terms containing x to the left-hand side (usually) and all terms not containing x to the right, changing the sign of any term which has to be removed from one side to the other." And yet the author has himself, two pages earlier, been teaching on the only rational plan, the application of the axioms that if equals be added to / taken from equals, the sums / remainders are equal. And again, what is the value of: "Change (mentally) the sign of the part to be *subtracted* and then add algebraically." All these appeals to some kind of black magic, *e.g.* "Change side, change sign," should be for ever swept away from our text-books. They are not even "memoria technica." Apart from this, Mr. Farquharson's book is worthy of consideration. It has, moreover, sets of revision exercises and examination papers for the three terms it is designed to cover.

Revision Papers in Arithmetic. By W. G. Borchardt. Pp. 156 + xxxii. 2s. 1917. (Rivingtons.)

This is a useful collection of well-graded examples in 100 papers of 7 questions each, followed by special exercises of many questions on Mensuration, Percentages, Decimals, Stocks respectively, with a final batch of 149 miscellaneous problems. Such collections are always useful, and may be used with any text-book in Arithmetic. This set is on all fours with the compiler's set of *Revision Papers in Algebra*, which was published with (2s.) and without Answers (1s. 6d.). The copy before us contains the Answers and is published at 2s.

Practical Arithmetic and Mensuration. By F. M. and C. H. Saxelby. Pp. vii+167. 2s. 6d. 1917. (Longmans, Green.)

This is a course for all pupils whose "main interest lies or will lie rather in the future in *making* things than in *buying* or *selling* them." For that purpose it seems admirably adapted. The slide rule is much in evidence. It is worth noting that De Morgan and Prof. Perry alike were convinced that the quickest way to master a "sliding rule" is to play with it, *i.e.* to solve all the problems the rule suggests by random positions of the scales. The striking effect produced by appeals to chance was perhaps first recognised by the famous Welsh genius, Robert Recorde, who used to startle his friends by difficult questions, the true results of which he worked out from the chance answers of "suche children or ydeotes as happened to be in the place." On the first page, the model answer to "find the prime factors of 32340" begins with 2, 3, 5, ... Surely it is better to try to turn 11, 10, 9.... The simple tests of divisibility by 2, 3, 9 are given, but those for 11, 8, 4 are omitted. The selection of examples is very interesting, and should suggest to the secondary school teacher and the examiner many useful novelties.

A Pocket Book for Chemists. By T. Bayley. Edited by R. Ensoll. Eighth edition. Pp. xvi+425. 7s. 6d. net. 1917. (E. Spon & Co.)

The mathematical tables, etc., in this volume fill about one-eighth of the book. Logarithms and anti-logarithms, vulgar fractions with decimal equivalents (one page), squares, cubes, square and cube roots, take eleven pages. Then, according to table of contents, come values of n^π and $\frac{\pi}{4}n_2$ for numbers from 1 to 100, which turn out to be tables for $n\pi$ and $\frac{\pi}{4}n^2$. Tables of capacity in imperial gallons of cylinders 1 in. high take pp. 16 to 22. The tables of conversion are for grammes into grains, percentage into lbs. per cwt., and cwts. and lbs. per ton; grains into grammes; millimetres into inches; inches, and fractions and decimals of an inch, into millimetres; lb./inch^2 into kg./cm^2., and kg./m^3. into lb./ft^3.

THE LIBRARY.

Change of Address.

The Library is now at 9 Brunswick Square, W.C., the new premises of the Teachers' Guild.

The Librarian will gladly receive and acknowledge in the *Gazette* any donation of ancient or modern works on mathematical subjects.

A copy of Prof. A. N. Whitehead's *Organisation of Thought* has been added to the Library.

Scarce Back Numbers.

Reserves are kept of A.I.G.T. Reports and Gazettes, and, from time to time, orders come for sets of these. We are now unable to fulfil such orders for want of certain back numbers, which the Librarian will be glad to buy from any member who can spare them, or to exchange other back numbers for them:

Gazette No. 8 (very important).
A.I.G.T. Report No. 11 (very important).
A.I.G.T. Reports, Nos. 10, 12.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.